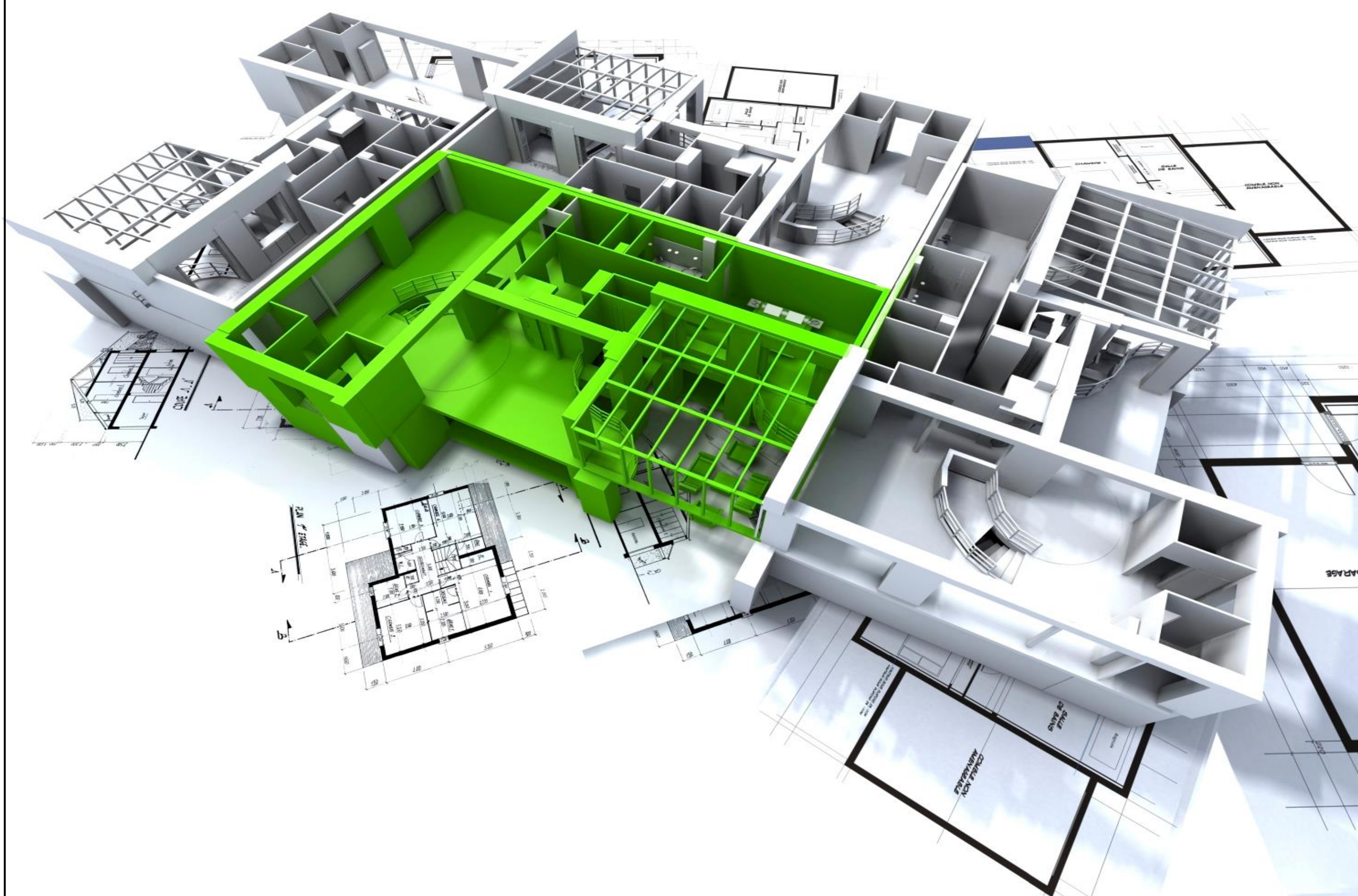


FLUIDS 2

BASIC FLUID DYNAMICS

In this section we will cover the laws which govern moving fluids – mass, energy and momentum conservation.

FLUIDS 2

BASIC FLUID DYNAMICS

OVERVIEW

In this unit you'll learn about:

- The role of conservation laws in fluids.
- Mass conservation and the continuity equation
  - Mass flow-rate
  - Volumetric flow-rate
  - Speed of flow in pipes
- Energy conservation and Bernoulli's equation
  - Pressure in moving fluids
  - Relationship between pressure and energy
  - The basic ideas of airfoil lift
  - The concept of airfoil driven turbines
- Momentum conservation and forces in fluids
  - The concept of thrust and jet forces
  - Fluid propulsion systems
  - The concept of momentum driven turbines
  - Forces in pipes

ASSUMED KNOWLEDGE FOR THIS SUBJECT

In this class it is assumed that you already have a knowledge of the following topics:

- *Basic fluid definitions – What a fluid is; The molecular nature of Fluids; Compressibility; Ideal and real fluids.*

- *Fluid parameters – Density, Pressure and Viscosity.*

- *Fluid Statics – Hydrostaic pressure and Buoyancy, etc*

OBJECTIVE

Fluid dynamics is a core part of engineering. It governs flow in pipes, channels, hydrodynamics and aerodynamics

TOPIC 1 – INTRODUCTION: FLUID CONSERVATION LAWS

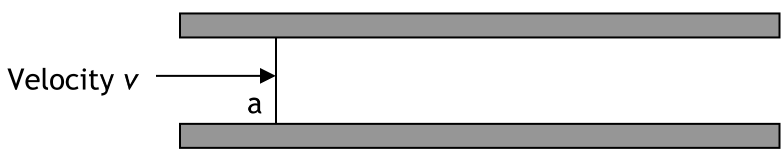
In this subject we concentrate on *Fluid Dynamics* - that is the science of fluid movement. We will cover three areas which are considered the basis of all fluid dynamics:

1. The Continuity Equation – A result of the conservation of mass in a fluid
2. Euler's and Bernoulli's Equations – A result of the conservation of energy in a fluid
3. Fluid Momentum – A result of the conservation of momentum

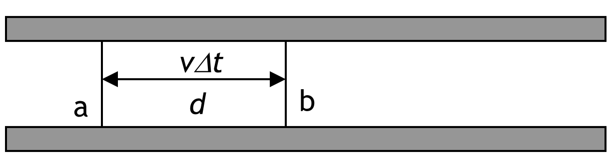
The Continuity Equation relates the speed of a flowing fluid, in a pipe or similar bounded space, to its area of flow. Euler's and Bernoulli's Equation's relate the velocity of the fluid to its pressure (and energy). Finally, Momentum relates a fluid's mass and velocity to the force that it generates.

TOPIC 2 – THE CONTINUITY EQUATION

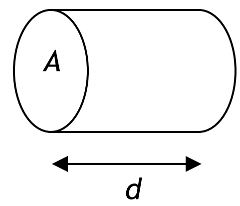
Consider an invisid flow along a pipe, as shown below.



The distance moved by an advancing front of fluid in time Δt (like that labelled "a" in the diagram) is $d = v\Delta t$. The fluid moves along to point "b."



The volume swept out in this time is obviously $Volume = d \times A$, where A is the area of the pipe



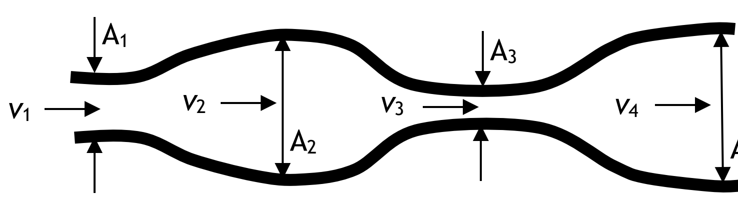
So, the mass contained in this volume is $\rho \times v \times \Delta t \times A = m$.

The mass flow rate $\dot{m}$ (that is, how many Kg of fluid per second are flowing) is $\frac{\Delta m}{\Delta t}$, so:

$$\frac{\Delta m}{\Delta t} = \rho v A$$

Now, let's think about this. If we're talking about fluid moving through a pipe, nothing can escape – it can't leak out the sides. So the mass of fluid in the pipe can't change – it's constant. This is why the continuity equation represents the conservation of mass. Although we're discussing a pipe in this example, the same argument also applies to the fluid moving between the stream-lines (see below) of an open or external flow.

This means that the *mass flow rate* is constant too (providing the flow entering the pipe is constant and no-one turns off the tap!). So <u>ρvA is constant in every place in the pipe</u> – and further, if we are talking about incompressible flows (liquids and slowly moving gasses), where ρ is constant, then vA is constant.



$$A_1v_1 + A_2v_2 + A_3v_3 + A_4v_4 = Constant$$

This means that, as the area gets smaller, the speed must increase to push all the mass through. You can see this by restricting the area of flow of a garden hose by putting your finger over the nozzle.

This idea of conservation of mass is one of the most important and basic principles of fluid mechanics and is summed up in the continuity equation.

Continuity Equation:

$\boxed{\rho Av = constant}$ For general invisid fluids

$\boxed{Av = constant}$ For incompressible fluids (liquids and slow gasses)

These two parameters are also used to specify flow-rate in the specifications of many systems:

- As already discussed ρvA is the *mass flow rate* – the flow in Kg/s – the weight of fluid per second.
- vA is the *volumetric flow rate* – the flow in m^3 – the volume of fluid per second (also often stated in litres per unit time).

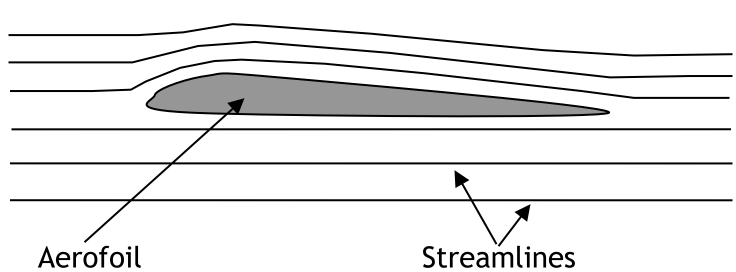
TASK 1

Water is moving through a circular pipe of diameter 2cm in diameter at 20cm/s.

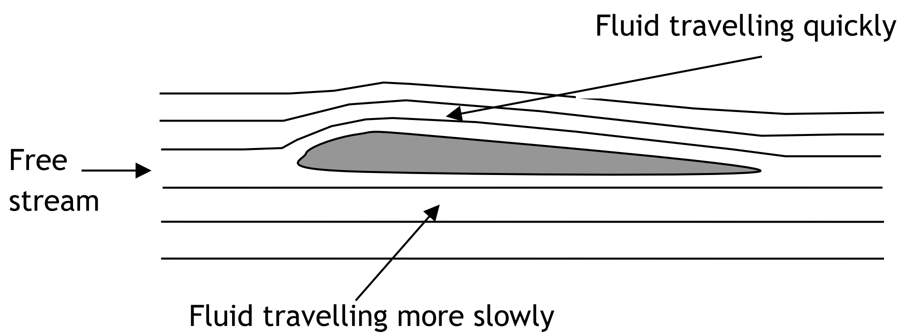
a. What is the volumetric flow rate in litres/s?
b. What is the mass flow rate?
c. If the pipe constricts to 1cm diameter, what is the speed of the flow?

Although we've talked about flow in pipes (called "internal flows") up until now, the continuity equation has wider applications. Let's consider these now.

If we release smoke into moving air or ink into moving water, it forms lines as it is swept along – these are called *Streamlines*. The streamlines flow around a smooth body (which is special shape called an *airfoil* or aerofoil*)* as shown in the diagram below.



The lines don't cross and therefore the fluid flows in-between them, rather like the fluid travelling down the pipe - so the continuity equation also applies here. Where the lines are close together (at the top of the airfoil) fluid is travelling fast (the closer together, the faster, like fluid flowing through a narrow pipe). Underneath the aerofoil, the lines are far apart and the fluid is travelling more slowly.



The reason that the air "sticks" to the top of the aerofoil like this, is due to its viscosity (which, after all, is the "stickiness" of the fluid). A fluid will tend to "stick" or be retarded by any smooth shape like this (this is called the "Coanda effect"). If we modelled the fluid as having no viscosity (as an invisid flow) – which you'll remember is a common simplification, we'd get completely the wrong answer (because the fluid wouldn't "follow" the shape of the aerofoil). As we'll see later, this is very important because without this shape aeroplanes wouldn't fly and turbines wouldn't turn!

TOPIC 3 – THE BERNOULLI EQUATION

The continuity equation related the velocity of the fluid (and its density in compressible flow) to changes in the flow area. But what about the other important fluid characteristic – pressure? The relationship between velocity, density and pressure is given by *Euler's Equation*. Here is a simple derivation, starting with Newton's second law of motion:

$$F = ma \qquad \text{- Newton's second law}$$

Let's divide both sides of this equation by a unit "box" volume of fluid $dxdydz$:

$$\frac{F}{dxdydz} = \frac{m}{dxdydz}a$$

but *dydz* is an area, so $\frac{F}{dydz} = \frac{F}{Area} =$ Pressure. Also $\frac{m}{dxdydz} = \frac{m}{volume} = \rho$. So:

$$\frac{p}{dx} = \rho a$$

but $a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v$ using the chain rule. So:

$$\frac{p}{dx} = \rho \frac{dv}{dx} v$$

and finally:

Euler's equation: $$\boxed{dp = -\rho v dv}$$

In deriving Euler's equation we assumed that the fluid was frictionless – so the equation only applies to invisid flows. You can see however, that the equation highlights and important point:

- *Pressure decreases with increasing velocity (assuming everything else (density) is constant).*

This point has important consequences (as we'll see later). Now, although Euler's equation is very important, it involves differentials, so we'd need to integrate it each time we used it. However, fortunately we can derive another extremely important equation from it by integrating it, just once, assuming constant density – Bernoulli's equation. So let's derive this now. Integrating Euler's equation:

$$\int_{p1}^{p2} dp = -\rho \int_{v1}^{v2} v dv$$

I won't bore you with the intermediate steps (you can do them as a homework problem), but the result is:

$$p_1 + \rho \frac{v_1^2}{2} = p_2 + \rho \frac{v_2^2}{2}$$

In other words, in a flow $p+\rho\frac{v^2}{2}$ is a constant. Now, this is actually a slightly simplified version which neglects the force of gravity. The more usual version is:

Bernoulli's equation: $\boxed{p+\frac{\rho v^2}{2}+\rho g h = \text{constant}}$

Bernoulli's equation has advantages over Euler's equation - it doesn't involve differentials and its uses parameters which are easy to measure. Remember through that it *only applies to invisid, incompressible flows.*

Can you see what Bernoulli's equation actually is? It's actually a statement of *energy conservation*. The total Kinetic and Potential Energy in a system is a constant (because we're working with a fluid there is also energy due to pressure as well).

$$p + \frac{\rho v^2}{2} + \rho g h = \text{Constant}$$

p	$\frac{\rho v^2}{2}$	$\rho g h$	Constant
Energy due to pressure	*Kinetic Energy*	*Potential Energy*	*Total Energy*

Compare this with another well known energy equation:

$$\frac{mv^2}{2} + mgh = \text{Constant}$$

$\frac{mv^2}{2}$	mgh	Constant
Kinetic Energy	*Potential Energy*	*Total Energy*

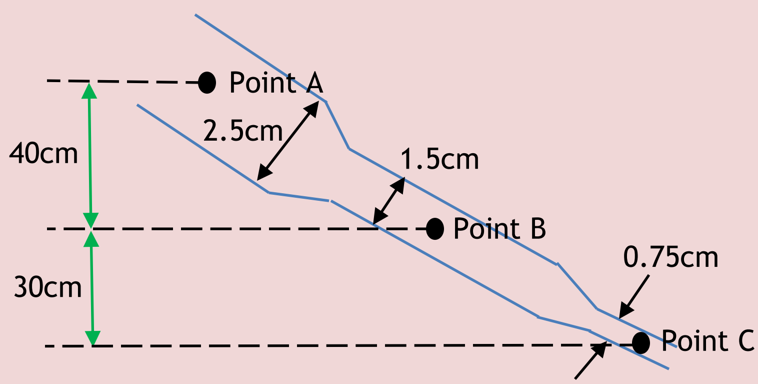
See the Similarity? So even if we hadn't derived Bernoulli from Euler. We could have worked it out from simple physics by writing down an equation to give the total energy in the fluid. All it says is that *the sum of all the energies of the fluid is a constant*.

In actual fact, the terms in the Bernoulli's equation aren't quite energy – you can see what they are if you compare mass (m) with density (ρ). Since $\rho = \frac{m}{volume}$ (that is to say, density is mass per unit-volume), the terms in Bernoulli's equation are energy per unit-volume (energy/m^3) – this makes complete sense if you think that we are discussing fluids (which unlike a solid object, can't always be easily isolated at a single point space).

The three pressure (energy density) terms should be familiar to you from first year as: The hydrostatic pressure ρgh due to the fluid weight, the static pressure p and the pressure associated with movement - dynamic pressure $\frac{\rho v^2}{2}$ in the fluid. The *Total Pressure* in the fluid being *Static Pressure* + *Hydrodynamic Pressure* + *Dynamic Pressure*.

TASK 2

1. *Look back at the pipe in task 1 – if the pressure at the inlet (the 2cm section) is 1.2 bar, what is pressure in the 1cm section (assuming the pipe is horizontal)?*
2. *Consider the section of circular pipe shown below:*

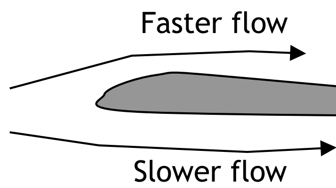


Water at point A is travelling at 10 cm/s at a static pressure of 1.5 bar. What is the velocity and static, dynamic and total pressures at points B and C?

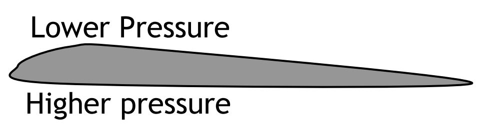
Now let's examine some of the applications of these ideas.

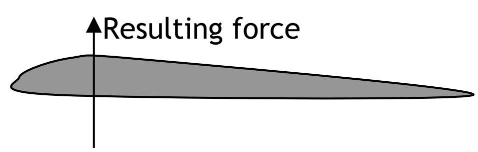
A – Lift on an Aerofoil wing (at low speed)

We are now in a position to put the final piece of the jigsaw in place with regard to flow over an aerofoil. We have already established, from the continuity equation, that flow is faster over the front-top surface. But what does this mean in terms of Bernoulli's equation.



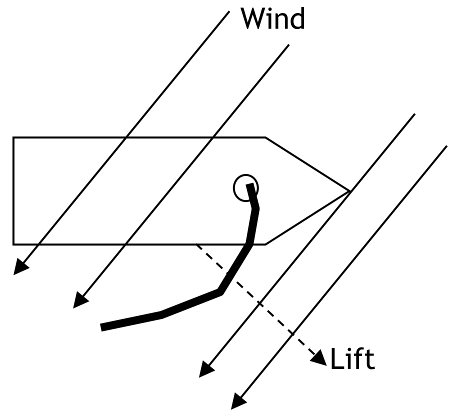
Well neglecting gravity (everything's at a constant height anyway), $p+\rho\frac{v^2}{2}$ is a constant between the stream-lines and because we're dealing with slow flows, ρ is a constant too. Therefore, as *v* rises *p* must fall. This is a general rule for invisid, incompressible flows as previously stated – all other things being constant - *when v rises p must fall*. This means that there is less pressure on the top surface of the aerofoil and more on the bottom, so there is an net upwards force – the aerofoil gets sucked up!





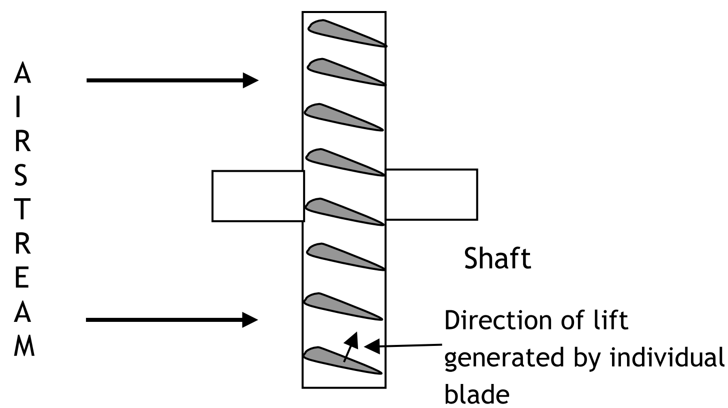
This is the reason why a wing provides lift. The actual calculation of lift for a wing is complex because we have to figure out the exact distribution of pressure across it (it becomes even more complex when we consider the 3-dimentional wing with length as well). Never the less, the principles outlined above, lie at the root of the phenomenon of flight.

The force generated by aerofoil shapes have many other applications. They are used underwater to stabilise ships and the sails of a sailing boat also work on this principle (they are not "pushed" along by the wind, except when it is directly behind them) – see diagram overleaf.

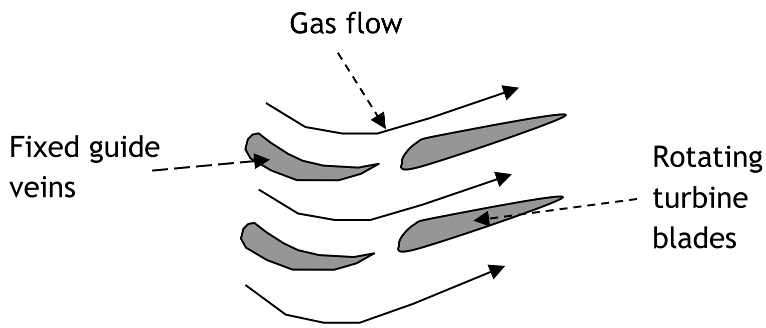


B – Turbines

Turbines are very important machines. They include Gas Turbines (jet engines), Water Turbines (renewable energy, hydroelectric schemes) and Air Turbines (wind power and other windmills). Most modern turbines use the aerofoil shape to generate rotational forces. The blades of the turbine are mini aerofoils, each of which contributes to the turning moment of the turbine. The diagram below shows the plan view of such a machine.



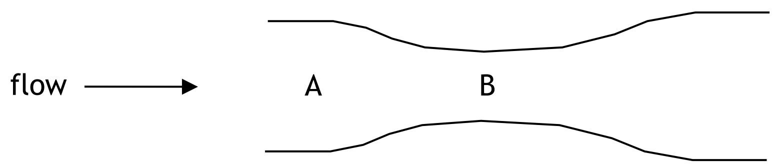
Sometimes turbines have stationary guide veins which direct the flow onto the turbine veins as shown in the diagram below.



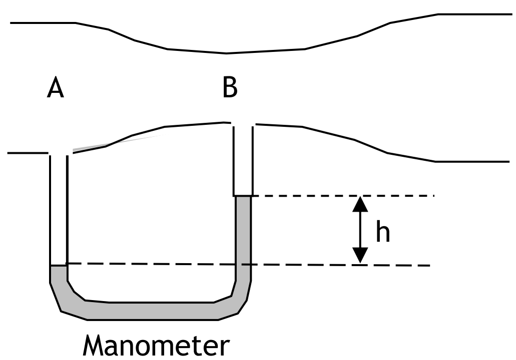
This arrangement is often seen in *axial compressors*. In a compressor, the mechanical energy is fed into the system from an external source, the object being to increase the gas pressure (so energy flow is the opposite way around from the turbine).

C – The Venturi

One final application of the continuity equation and Bernoulli's equation is the *Venturi Meter*. A Venturi is a pipe with a waisted constriction as shown below.



Obviously, at point B the speed of flow is higher than at point A and the pressure is therefore lower. This principle can be used to measure the flow rate, since the mass flow rate is proportional to the difference in pressure.



TOPIC 4 – FLUID MOMENTUM

Fluids are heavy – they have mass. They are also often moving at speed. Therefore, like a solid object, they have momentum. So when they hit something, they exert a force on it (just as any solid object, like a car, hitting something, exerts a force). This means that they also experience reactions forces according to Newton's Third Law of Motion - and when a nozzle or pipe spews out fluid, it displays a backwards reaction force (just as a rifle recoils in response to the momentum of a fired bullet).

In solid systems, we usually write momentum as *I=mv* (mass × velocity). However, as we've seen, in a fluid, stuff is flowing along all the time, so instead of mass, it's easier to use mass flow rate $\frac{dm}{dt}$ (usually written as $\dot{m}$). So our *mv* is better replaced by $\dot{m}v$. Note that this is the *rate of change of momentum*, rather than momentum itself.

However, rate of change of momentum is actually equal to force. You can see why if we change *v* rather than *m*:

$$m\dot{v} = m\frac{dv}{dt} = ma = F \qquad \text{Newton's Second Law}$$

acceleration

So:

Fluid Momentum $\boxed{F = \dot{m}v}$

So a *force generated by a fluid is its velocity times its mass flow rate*. At this point, it's worth mentioning something which may cause confusion. Some books refer to Euler's equation or the Bernoulli equation as the "Momentum Equation" (because you can also derive them from a consideration of fluid momentum). This is why I've referred to this equation as "Fluid Momentum" rather than the "Momentum Equation."

TASK 3

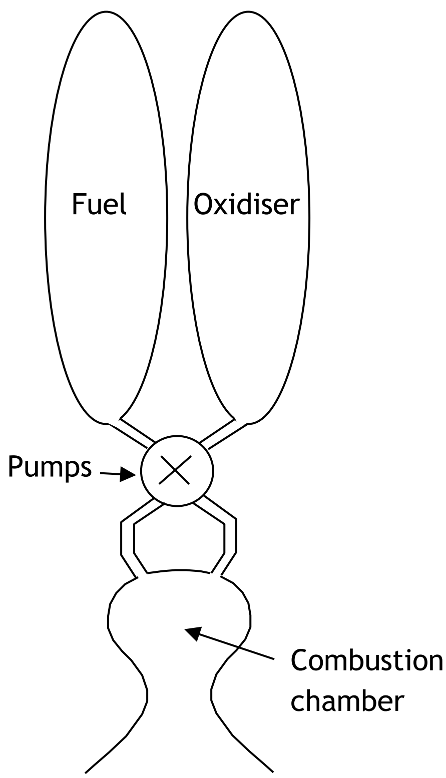
Water is moving through a circular hose of diameter 1cm in diameter at 20cm/s. What is the force on the hose.

Let's look at some applications of this. These will help to illustrate its usefulness.

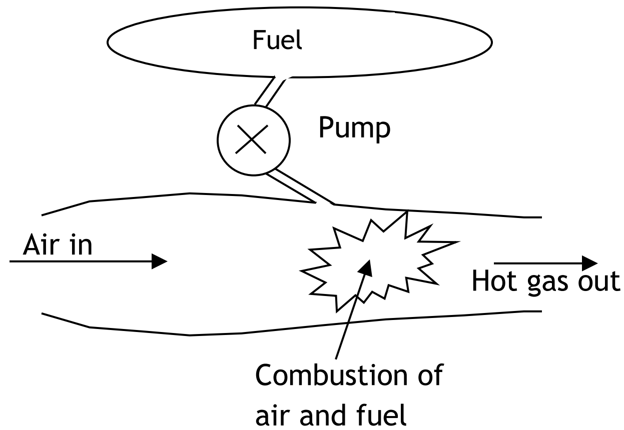
A – The jet engine and the rocket engine

Both these engines are known as *Reaction Engines* because their developed trust is a “reaction” to the momentum of gas rushing out of them. Diagrams showing their operation are given below.

Rocket Engine



Jet Engine



$Thurst = \dot{m}v = (\dot{m}_{fuel} + \dot{m}_{oxidiser})v_e$

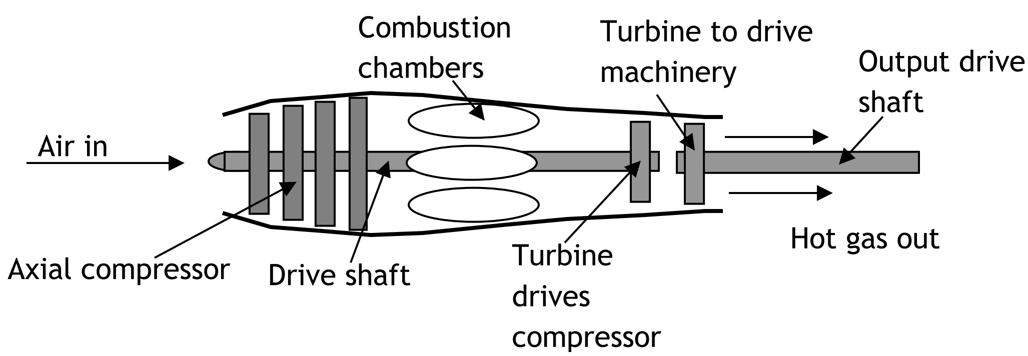
$Thurst = \dot{m}v = (\dot{m}_{air} + \dot{m}_{fuel})v_e - \dot{m}_{air}v_{in}$

Where v_e is the exhaust velocity and v_{in} is the velocity of the air entering the jet.

In the jet engine, air is usually supplied by the atmosphere. It is then compressed. Fuel is added and the resulting mixture burnt in a combustion chamber. The hot gas generated by the burning mixture expands rapidly through a nozzle, generating a reaction force. As it expands it drives a turbine which in turn drives the compressor. If the engine is being used to power something else (for example a power station or a propeller) a second turbine extracts power from the exhaust for this.

TASK 4

In the F1 rocket engine (the engine which powered Apollo 11 to the moon) used 58560 liters per minute of Kerosene (paraffin) fuel and 93920 liters per minute of liquid oxygen oxidizer. The exit velocity of the exhaust 2989 m/s. Calculate the engine thrust.

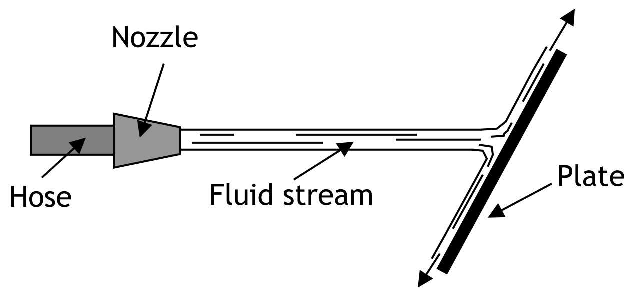


Note that this drawing is just a representation and not to scale.

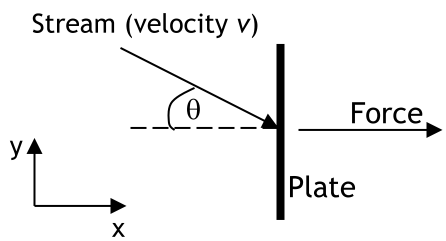
The rocket is, in effect, a jet which carries its own air (because there’s none in space!)

B – Force of a fluid stream hitting an object

Suppose that we have a fluid stream hitting an object – what force does it generate? Consider the diagram below.



It's probably easier to see what's happening if we tilt the plat so that it's vertical.



If we are trying to find the force pushing the plate straight back (force F_x), then we need to find the stream velocity in the *x* direction. $\cos\theta = \frac{v_x}{v}$ or $v_x = v\cos\theta$. We already know that the force is $F = \dot{m}v$, so:

$$F_x = \dot{m}v\cos\theta$$

We can simplify this further, because according to the continuity equation, $\dot{m} = \rho AV$, so:

$$F_x = \rho A v^2 \cos\theta \quad \text{(A = cross sectional area of stream)}$$

or, if the stream hits the plate straight on: $\cos\theta = 1$ (because $\theta = 0$) and:

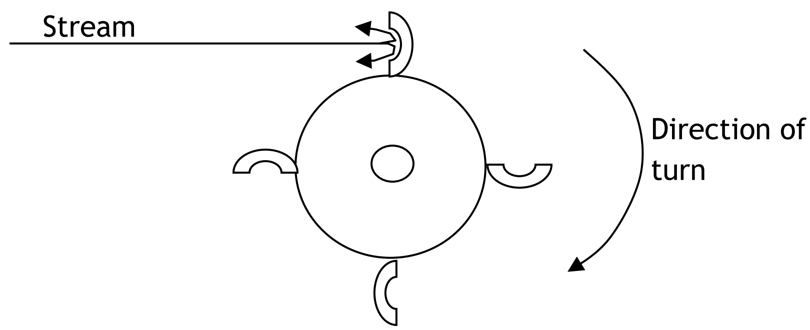
$$F = \rho A v^2$$

Or if the plate is moving at speed *u* away from the stream:

$$F_x = \rho A \cos\theta (v-u)(v-u)$$

Prove this later as homework.

One application of this is the *Pelton Wheel*. This is a primitive turbine, which works not through aerofoil lift (like many other turbines) but by the impact of a jet of fluid. Depending on the type of fluid used and its speed, impact turbines can be more efficient than aerodynamic ones – and it is also possible to construct machines which combine both effects.

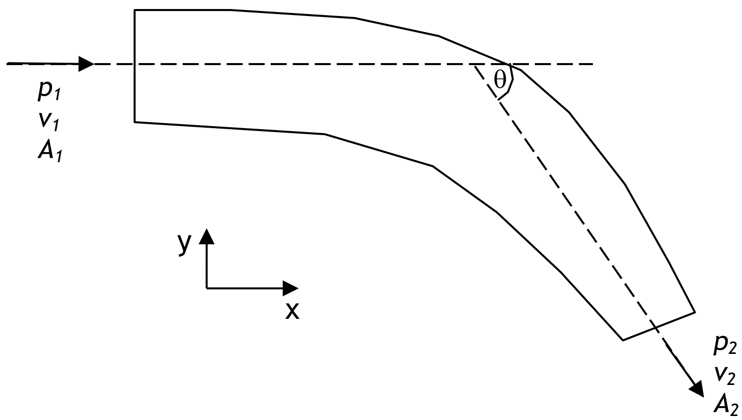


TASK 5

A flat plate sitting on a frictionless rail, has a mass of 1kg and is inclined at 30^0 to the horizontal. It is impacted by a horizontal jet of water of diameter 0.5cm and velocity 1m/s (note: it is leaning away from the water jet). What is the acceleration of the plate?

C – Forces due to bends in pipes

Consider the pipework shown below. Because the fluid has momentum and it is being forced to change direction, it exerts a force on the bend in the pipe. The force comes from two basic sources. Firstly, the change of momentum of the fluid as mentioned above; secondly, from the change of pressure between inlet and outlet. Because pressure is force / area, a force is exerted when the pressure changes. Actually, this force is also present in some of the previous examples given (like the jet engine) but I ignored it because it was generally small in these cases.



The pipe has been rotated until its inlet is parallel with the *x* axis. To calculate the total force on the pipe we'll split the two forces into *x* and *y* directions:

Force in the x direction $\Rightarrow F_x = F_{xp} - \dot{m}(v_{out} - v_{in})_x$

Force in the y direction $\Rightarrow F_y = F_{yp} - \dot{m}(v_{out} - v_{in})_y$

Force due to change in pressure

Force due to change in momentum

Let's deal with the pressure forces first:

$$F_{xp} = p_1 A_1 - p_2 A_2 \cos\theta$$

x direction in line with inlet

$$F_{yp} = 0 - p_2 A_2 \sin\theta$$

Now the forces due to momentum changes:

$$\dot{m}(v_{out} - v_{in})_x = \rho A_1 v_1 (v_2 \cos\theta - v_1)$$

$$\dot{m}(v_{out} - v_{in})_y = \rho A_2 v_2 (-v_2 \sin\theta)$$

So now the whole expression:

$$F_x = p_1 A_1 - p_2 A_2 \cos\theta - \rho A_1 v_1 (v_2 \cos\theta - v_1)$$

$$F_y = p_2 A_2 \sin\theta + \rho A_2 v_2^2 \sin\theta$$

From these two pieces of information, you can work out the total resultant force on the pipe F_t and the angle ϕ of this force, relative to the x axis:

$$F_t = \sqrt{F_x^2 + F_y^2}$$

$$\phi = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

SUMMARY

- The main equations used in fluid dynamics are derived from the conservation of Mass, Energy and Momentum.
- The Continuity Equation comes from the conservation of mass and is used to calculate the velocity of a fluid. It comes in two forms – one of which only applies to incompressible fluids, the other applies more generally.
- Two common measures of flow are mass flow-rate and volumetric flow-rate – these are closely related to the Continuity Equation.
- The continuity equation also applies to flow between stream-lines in external flow situations.
- The Bernoulli Equation comes from conservation of energy and is (most often) used to calculate the pressure in a fluid.
- The Bernoulli Equation only applied to incompressible, inviscid flows.
- Pressure is energy per unit volume.
- The Total Pressure in a fluid = Static Pressure + Dynamic Pressure
- Flow is faster over the top of an airfoil resulting in lower pressure.
- Venturi meters measure flow-rate though pressure changes.
- Many turbines are based on forces generated by airfoil lift.
- Momentum is conserved in fluids as in solids.
- Thrust can be calculated from mass flow rate and velocity.
- Forces on plates and pipes can be calculated from changes in momentum.